the analyst's intuition concerning the effectiveness of using boundary restraint to greatly increase buckling loads must be drastically modified when dealing with transversely isotropic materials.

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Film Cooling Effectiveness with Helium and Refrigerant 12 Injection into a Supersonic Flow

R. J. Goldstein* and M. Y. Jabbari†

University of Minnesota, Minneapolis, Minn.

Nomenclature

c_p = specific heat
M = injection parameter, $\rho_2 U_2/\rho_\infty U_\infty$
Pr = Prandtl number
Re_2 = slot Reynolds number, $\rho_2 U_2 s/\mu_2$
s = length of porous section in flow direction
T = temperature
T_r = recovery temperature
T_* = reference temperature, [Eq. (2)]
U_∞ = velocity of mainstream in direction along the wall
U_2 = velocity of injectant at point of injection normal to the wall
W = molecular weight
x = distance along the wall downstream from the point of injection
ρ = density
μ = dynamic viscosity
η_{is} = high-speed flow effectiveness based on isoenergetic flow temperatures, [Eq. (1)]
ξ_* = dimensionless parameter, [Eq. (6)]

Subscripts

0 = condition in the stagnation chamber
2 = properties of injectant at point of injection
∞ = properties of mainstream
aw = adiabatic wall
$(\ )_{is}$ = designates isoenergetic flow condition
$*$ = evaluated at reference temperature T_*

EXPERIMENTAL measurements are reported for the film cooling effectiveness with normal injection (through a porous strip) of helium and refrigerant 12 into a two-dimensional turbulent boundary layer of a Mach 3 airflow. Related work has been reported in Refs. 1–4 (analytical studies) and Refs. 4–9 (experimental studies).

The apparatus has been described elsewhere.[4] The main flow of air has a Mach number of approximately 3 and a total temperature of about 295 K. The secondary flow (either helium or refrigerant 12) is subsonic and injected through a narrow porous section with an injection parameter $0.0018 < M < 0.0115$ and temperature $228\text{ K} < T_2 < 355\text{ K}$. A simple sketch of the geometry is included in Fig. 1.

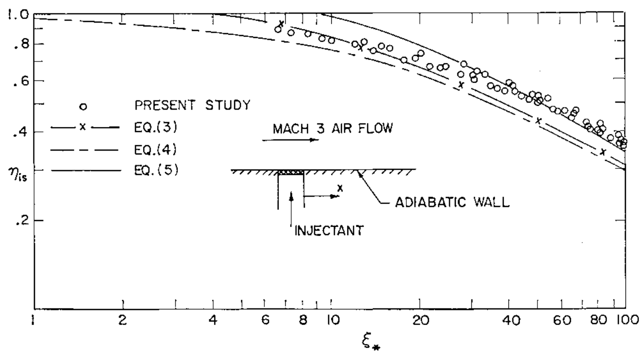

Fig. 1 Film cooling effectiveness with injection of helium for $M = 0.0018 - 0.0034$, $T_2 = 228 - 335°K$ {taking $c_{p\infty}/c_{p2} = 0.194$, and $Pr_{\infty *} = 0.728$ and $Re_2\ \mu_2/\mu_{\infty *} = 385$ [used in Eq. (5)]}.

Following earlier studies,[4,8] supersonic film cooling effectiveness is defined using isoenergetic flow temperatures

$$\eta_{is} = \frac{T_{aw} - (T_{aw})_{is}}{T_2 - (T_2)_{is}} \tag{1}$$

(Isoenergetic flow conditions are obtained with the same mainstream flow when the secondary gas, at the same rate of injection, has a total temperature equal to that of the main flow.) In order to eliminate the effect of day-to-day variation of the main-flow total temperature, the effectiveness is computed as[8]

$$\eta_{is} = \frac{(T_{aw}/T_0) - (T_{aw}/T_0)_{is}}{(T_2/T_0) - (T_2/T_0)_{is}} \tag{1a}$$

A reference state[4] is defined using

$$T_* = 0.28T_\infty + 0.72T_r \tag{2}$$

Modification of the earlier low-speed flow analyses to include effects of compressibility and foreign gas injection leads to the following equations for supersonic flow.[4,5,10] Stollery and El-Ehwany[2]

$$\eta_{is} = \{1 + (c_{p\infty}/c_{p2})[0.33\xi_*^{0.8} - 1]\}^{-1} \tag{3}$$

Kutateladze and Leont'ev[1]

$$\eta_{is} = \{1 + (c_{p\infty}/c_{p2})[0.33(4 + \xi_*)^{0.8} - 1]\}^{-1} \tag{4}$$

Goldstein and Haji-Sheikh[3]

$$\eta_{is} = \frac{1.9(Pr_{\infty *})^{2/3}}{1 + 0.33(c_{p\infty}/c_{p2})\xi_*^{0.8}(1 + \beta)} \tag{5}$$

with

$$\beta = 0.00015[\mathrm{Re}_2(\mu_2/\mu_{\infty *})](W_\infty/W_2) \tag{5a}$$

The parameter ξ_* in the foregoing equations is defined as

$$\xi^* = (x/Ms)[Re_2(\mu_2/\mu_{\infty *})]^{-0.25}(\rho_{\infty *}/\rho_\infty) \tag{6}$$

In Fig. 1 the experimental film cooling effectiveness for helium injection is plotted against the parameter ξ_*. On the same graph are shown the modified theoretical models. Data for refrigerant 12 injection are compared with the theoretical models in Fig. 2. Data using air injection were also obtained,[10] and the agreement of these with an earlier study[4] is satisfactory.

As indicated on the two graphs, the definition of effectiveness using isoenergetic flow temperatures and evaluation of the fluid properties at the reference temperature correlate the

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* Professor, Department of Mechanical Engineering.

† Research Assistant, Department of Mechanical Engineering.

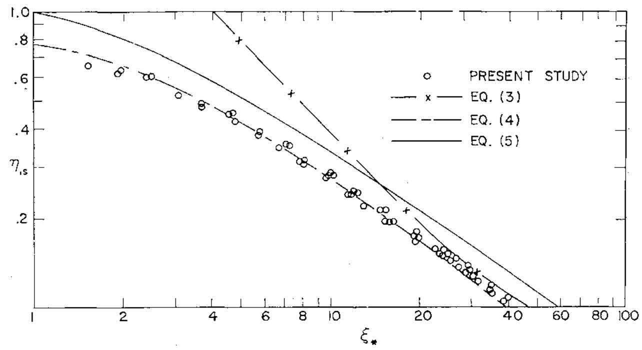

Fig. 2 Film cooling effectiveness with injection of refrigerant 12 for $M = 0.0082 - 0.0115$, $T_2 = 328 - 355°K$ {taking $c_{p\infty}/c_{p2} = 1.58$, and $Pr_\infty* = 0.728$ and $Re_2\ \mu_2/\mu_\infty* = 1290$ [used in Eq. (5)]}.

present data and provide reasonable agreement with the modified theoretical models of subsonic film cooling. The model of Kutateladze and Leont'ev [Eq. (4)], although not realistic in its assumptions of uniform mixture and temperature across the boundary layer, predicts the experimental data obtained with refrigerant 12 injection excellently (Fig. 2). This model also predicts the air injected data,[4,10] but fails to predict adequately the data for helium injection. The helium injection data are approximated better by Eq. (5) which had been used to predict subsonic film cooling effectiveness with air and helium injection.[3] Shadowgraph observations indicate that the composition of the injected gas affects the boundary-layer thickness. This effect is included in Eq. (5) by the molecular weight ratio in parameter β. Rederiving Eq. (4) to include this effect on boundary-layer thickness gives better agreement with the data for helium injection, but the new equation, besides being complicated, does not adequately predict the results of refrigerant 12 injection.

One can thus conclude that there is relatively good agreement between the experimental data for supersonic film cooling and modified theoretical models of subsonic film cooling when a foreign gas is injected through a porous section into the main flow.

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Undestructive Determination of the Buckling Load of an Elastic Bar

Menahem Baruch*
Georgia Institute of Technology, Atlanta, Ga.

Introduction

It was always important to find some practical method of determining the buckling load of a bar without destroying it. The famous Southwell[1] plot is a very important one, but it requires loading in the direction of the bar's axis, and the load must approach the critical value. Another method recently was proposed by Horton and Struble.[2] In this method the load is applied in a direction perpendicular to the bar's axis. By measuring the deflections or the angles of rotation of the deflected bar, it was found that there is a connection between the buckling load and these two parameters. The method works well in some cases but not so well in other cases, especially when one end is laterally restricted by an elastic spring. In this case the method ceases to be exact enough. The reason why the proposed Horton and Struble method fails in some cases may be as follows. (See Figs. 1 and 2.)

For the two structural systems seen in Figs. 1 and 2, the bars and the springs at the ends are the same, but the boundary conditions are different. When at least one of the two lateral spring constants (β_1, β_2) is zero or finite, the buckling force P enters into the boundary conditions in Fig. 1. Therefore, the respective boundary conditions in the two cases are different. This is probably the reason why the method proposed in Ref. 2 fails when the bar is supported on lateral springs. The Southwell plot works well because the boundary conditions are fulfilled all the time.

The boundary conditions in Figs. 1 and 2 are not the same, but the spring constants (α_1, α_2, β_1, β_2) are identical. Based on this fact, another method which is more general and will work in all the cases in the elastic region is proposed here.[3] By loading the bar as in Fig. 2 and measuring deflections and rotations it is possible to obtain data from which to calculate the spring constants and then to calculate the buckling load.

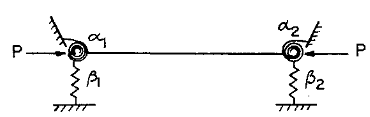

Fig. 1 Spring constrained beam loaded by axial force *P*.

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* Visiting Professor in the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Ga. On leave from Technion, Haifa, Israel. Member AIAA.